

Logarithmic Functions

Lecture 35
Section 4.2

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Wed, Mar 29, 2017

Reminder

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- Be there.

Objectives

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- Learn (or review) the properties of logarithmic expressions.
- Learn the properties of logarithmic functions.

Logarithms

Definition (Logarithm)

Let $b > 0$ and let x and y be real numbers. If

$$b^x = y,$$

then x is the **base- b logarithm** of y . We write this as

$$x = \log_b y.$$

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- **Change of base:** $\log_b x = \frac{\log_a x}{\log_a b}$.

Natural Logarithms

Definition (Natural Logarithm)

When the base of the logarithm is the natural base e , the logarithm is called the **natural logarithm** and the notation is

$$\ln x$$

instead of $\log_e x$.

Logarithmic Functions

Definition (Logarithmic Function)

A **logarithmic function** is a function of the form $f(x) = \log_b x$ for some base $b > 0$, $b \neq 1$.

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Example 4.2.12 (modified)

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- (c) At what annual rate, if compounding is continuous?
- (d) If her investment is compounded monthly at 6%, how long would it take to reach \$4,000?