Derivatives of Exponential and Logarithmic Functions

Lecture 36
Section 4.3

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Objectives

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- Be able to differentiate e^x and b^x.
- Be able to differentiate $\ln x$ and $\log_b x$.
- Work applications involving relative growth rates.

The Derivatives of e^x and b^x

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$$\frac{d}{dx}\left(e^x\right) = e^x.$$

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$$\frac{d}{dx}(e^x) = e^x.$$

$$\frac{d}{dx}(b^x) = b^x \ln b.$$

The Derivatives of $\ln x$ and $\log_b x$

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$$\frac{d}{dx}(\ln x) = \frac{1}{x}.$$

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Recall that the relative rate of change of a function $Q(x)$ is

$$\frac{Q'(x)}{Q(x)}.$$

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Recall also that

$$\frac{d}{dx}(\ln Q(x)) = \frac{Q'(x)}{Q(x)}.$$

Example

Example 4.3.14

A country exports three goods, wheat W, steel S, and oil O. Suppose that that at a particular time T, the revenue, in billions of dollars, derived from each of these goods is

$$W(t) = 4 \quad S(t) = 7 \quad O(t) = 10$$

and that S is growing at 8%, O is growing at 15%, while W is declining at 3%.

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(a) At what relative rate is total export revenue growing (or declining) at this time?

Example

Exercise 4.3.76:

The national income $I(t)$ of a particular country is increasing by 2.3% per year, while the population $P(t)$ of the country is decreasing at the annual rate of 1.75%. The per capita income C is defined to be

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(a) Find the derivative of $\ln C(t)$.

(b) Use the result of part (a) to determine the percentage rate of growth of per capita income