Second-Order Partial Derivatives

Lecture 43
Section 7.2

Robb T. Koether

Hampden-Sydney College

Fri, Apr 14, 2017

Announcement

Announcement

- Test #4 is next Friday, April 21.

Announcement

Announcement

- Test #4 is next Friday, April 21.
- It will cover Chapters 4 and 7.

Announcement

Announcement

- Test #4 is next Friday, April 21.
- It will cover Chapters 4 and 7.
- Be there.

Objectives

Objectives

- Define the second-order partial derivatives (all four of them).
- Practice computing them.
- Interpret them graphically.

Second-Order Partial Derivatives

Definition (First-Order Partial Derivative)

The **first-order partial derivatives** of a function $f(x, y)$ are the two partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

Second-Order Partial Derivatives

Definition (First-Order Partial Derivative)

The **first-order partial derivatives** of a function $f(x, y)$ are the two partial derivatives $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$.

Definition (Second-Order Partial Derivative)

The **second-order partial derivatives** of a function $f(x, y)$ are the two partial derivatives of each of the two first-order partial derivatives.

Notation

Notation

- The partial with respect to x twice is denoted

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} \quad \text{or} \quad (f_x)_x = f_{xx}.$$

- The partial with respect to x followed by the partial with respect to y is denoted

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y\, \partial x} \quad \text{or} \quad (f_x)_y = f_{xy}.$$

Notation

Notation

- The partial with respect to y twice is denoted

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2} \quad \text{or} \quad (f_y)_y = f_{yy}.$$

- The partial with respect to y followed by the partial with respect to x is denoted

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x\,\partial y} \quad \text{or} \quad (f_y)_x = f_{yx}.$$

Practice

Practice

Find the four second-order partial derivatives of the following functions.

- $f(x, y) = x^4 y^2$

Practice

Practice

Find the four second-order partial derivatives of the following functions.

- $f(x, y) = x^4y^2$
- $f(x, y) = (2x + 3y)^3$

Practice

Practice

Find the four second-order partial derivatives of the following functions.

- $f(x, y) = x^4 y^2$
- $f(x, y) = (2x + 3y)^3$
- $f(x, y) = x^2 - 3xy - y^2$

Practice

Practice

Find the four second-order partial derivatives of the following functions.

- $f(x, y) = x^4y^2$
- $f(x, y) = (2x + 3y)^3$
- $f(x, y) = x^2 - 3xy - y^2$
- $f(x, y) = x \ln y$

Practice

Practice

Find the four second-order partial derivatives of the following functions.

- $f(x, y) = x^4 y^2$
- $f(x, y) = (2x + 3y)^3$
- $f(x, y) = x^2 - 3xy - y^2$
- $f(x, y) = x \ln y$
- $f(x, y) = e^{xy}$

Concavity

Concavity

- $\dfrac{\partial^2 f}{\partial x^2}$ indicates the concavity in the x direction.
- $\dfrac{\partial^2 f}{\partial y^2}$ indicates the concavity in the y direction.
- The "mixed partials" do not directly indicate concavity.

Example

Example

For the function

$$f(x, y) = 4x^2 - 3xy - y^3$$

find and analyze (max, min, conc, etc.)

- The curve of intersection with $y = 2$.
- The curve of intersection with $x = 1$.