

Optimizing Functions of Two Variables

Lecture 45
Section 7.3

Robb T. Koether

Hampden-Sydney College

Tue, Apr 18, 2017

Reminder

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- Be there.

Objectives

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- Locate critical points for $f(x, y)$.
- Classify each critical points as maximum, minimum, or neither.
- Learn the Second Partials Test.
- Applications.

Relative Extremes

Definition (Relative Maximum)

A **relative maximum** of a function $f(x, y)$ occurs at a point (a, b) if $f(a, b) \geq f(x, y)$ for all points (x, y) in some circular disk (sufficiently small) centered at (a, b) .

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Definition (Relative Minimum)

A **relative minimum** of a function $f(x, y)$ occurs at a point (a, b) if $f(a, b) \leq f(x, y)$ for all points (x, y) in some circular disk (sufficiently small) centered at (a, b) .

Critical Points

Definition (Critical Point)

A point (a, b) is a **critical point** of a function $f(x, y)$ if

$$f_x(a, b) = 0 \quad \text{and} \quad f_y(a, b) = 0.$$

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- If $f(x, y)$ has a relative extreme value, then it must occur at a critical point.

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- $f(x, y) = x^2 + y^2 - 4xy$

The Second Partial Test

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Let $f(x, y)$ be a function of x and y whose partial derivatives f_x , f_y , f_{xx} , f_{yy} , and f_{xy} all exist and let

$$D(x, y) = f_{xx}f_{yy} - (f_{xy})^2.$$

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- (1) Find all critical points of $f(x, y)$.
- (2) For each critical point (a, b) ,
 - (a) If $D(a, b) < 0$, then there is a **saddle point** at (a, b) .

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 - (i) If $f_{xx}(a, b) > 0$, then there is a **relative minimum** at (a, b) .
 - (ii) If $f_{xx}(a, b) < 0$, then there is a **relative maximum** at (a, b) .

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 - (i) If $f_{xx}(a, b) > 0$, then there is a **relative minimum** at (a, b) .
 - (ii) If $f_{xx}(a, b) < 0$, then there is a **relative maximum** at (a, b) .
 - (c) If $D(a, b) = 0$, then the test is inconclusive.

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