Constrained Optimization

Lecture 48
Section 7.5

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Objectives

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- Economic applications of the method of Lagrange multipliers.

An Example

Example 7.5.3:

Esteban has \$600 to spend on two commodities, the first of which costs \$20 per unit and the second \$30 per unit. Suppose that the utility her derives from x units of the first commodity and y units of the second commodity is given by the **Cobb-Douglas utility function**

$$U(x, y) = 10x^{0.6}y^{0.4}.$$

How many units of each commodity should Esteban buy to maximize utility?

Cobb-Douglas Functions

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- Let $f(x, y) = Ax^{\alpha}y^{\beta}$, where $\alpha + \beta = 1$.
- Let the constraint be $ax + by = k$.
- Maximize $f(x, y)$ subject to the constraint $ax + by = k$.

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- The solution turns out to be

$$x = \alpha\left(\frac{k}{a}\right),$$
$$y = \beta\left(\frac{k}{b}\right).$$

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This is Exercise 7.5.26.

Allocation of Funds

Exercise 7.5.22

A manufacturer is planning to sell a new product at the price of \$150 per unit and estimates that if x thousand dollars is spent on development and y thousand dollars is spent on promotion, approximately

$$\frac{320y}{y+2} + \frac{160x}{x+4}$$

units of the product will be sold. The cost of manufacturing the product is \$50 per unit. If the manufacturer has a total of \$8,000 to spend on development and promotion combined, how should this money be allocated to generate the largest possible profit?

Constant Elasticity of Substitution

Exercise 7.5.31

Use the method of Lagrange multipliers to maximize the constant-elasticity-of-substitution (CES) production function

$$Q = 55\left[0.6K^{-1/4} + 0.4L^{-1/4}\right]^{-4}$$

subject to the constraint

$$2K + 5L = 150.$$