The Interpretation of λ

Lecture 49
Section 7.5

Robb T. Koether

Hampden-Sydney College

Wed, Apr 26, 2017

Objectives

Objectives

- Interpret the meaning of the Lagrange multiplier λ.

Allocation of Funds

Exercise 7.5.22

A manufacturer is planning to sell a new product at the price of \$150 per unit and estimates that if x thousand dollars is spent on development and y thousand dollars is spent on promotion, approximately

$$\frac{320y}{y+2} + \frac{160x}{x+4}$$

units of the product will be sold. The cost of manufacturing the product is \$50 per unit. If the manufacturer has a total of \$8,000 to spend on development and promotion combined, how should this money be allocated to generate the largest possible profit?

The Interpretation λ

- Given the constraint $g(x, y) = k$, we could consider k to be a fourth variable (x, y, λ, k).

The Interpretation λ

- Given the constraint $g(x, y) = k$, we could consider k to be a fourth variable (x, y, λ, k).
- Then we can compute $\frac{\partial F}{\partial k}$, which will be λ.

The Interpretation λ

- Given the constraint $g(x, y) = k$, we could consider k to be a fourth variable (x, y, λ, k).
- Then we can compute $\frac{\partial F}{\partial k}$, which will be λ.
- Thus, λ represents the rate of change of $F(x, y)$ as the constraint is increased or decreased.

The Interpretation λ

- Given the constraint $g(x, y) = k$, we could consider k to be a fourth variable (x, y, λ, k).
- Then we can compute $\frac{\partial F}{\partial k}$, which will be λ.
- Thus, λ represents the rate of change of $F(x, y)$ as the constraint is increased or decreased.
- If x and y represent **money** and $F(x, y)$ represents **utility**, then λ is the **marginal utility of money**.

Marginal Analysis

Exercise 7.5.23

In Exercise 7.5.22, suppose that the manufacturer decides to spend \$8,100 instead of \$8,000 on the development and promotion of the new product.

Exercise 7.5.23

In Exercise 7.5.22, suppose that the manufacturer decides to spend \$8,100 instead of \$8,000 on the development and promotion of the new product.

(a) Use the Lagrange multiplier λ to estimate how this change will affect the maximum possible profit.

Marginal Analysis

Exercise 7.5.23

In Exercise 7.5.22, suppose that the manufacturer decides to spend \$8,100 instead of \$8,000 on the development and promotion of the new product.

(a) Use the Lagrange multiplier λ to estimate how this change will affect the maximum possible profit.

(b) Find the new values for x and y and verify the answer in part (a).

An Example

Example 7.5.3

Esteban has \$600 to spend on two commodities, the first of which costs \$20 per unit and the second \$30 per unit. Suppose that the utility he derives from x units of the first commodity and y units of the second commodity is given by the **Cobb-Douglas utility function**

$$U(x, y) = 10x^{0.6}y^{0.4}.$$

An Example

Example 7.5.3

Esteban has \$600 to spend on two commodities, the first of which costs \$20 per unit and the second \$30 per unit. Suppose that the utility he derives from x units of the first commodity and y units of the second commodity is given by the **Cobb-Douglas utility function**

$$U(x, y) = 10x^{0.6}y^{0.4}.$$

(a) We found that when $x = 360$ and $y = 80$, U had a maximum value of xxx.

An Example

Example 7.5.3

Esteban has \$600 to spend on two commodities, the first of which costs \$20 per unit and the second \$30 per unit. Suppose that the utility he derives from x units of the first commodity and y units of the second commodity is given by the **Cobb-Douglas utility function**

$$U(x, y) = 10x^{0.6}y^{0.4}.$$

(a) We found that when $x = 360$ and $y = 80$, U had a maximum value of xxx.

(b) Estimate the change in U if Esteban has an additional \$20 to spend.