Nonlinear Regression

Lecture 51
Section 7.4

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Objectives

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- Derive other least-squares equations.
- Apply the models to the DJIA.

Quadratic Regression

Quadratic Regression

Quadratic regression requires finding a quadratic function

$$y = ax^2 + bx + c$$

that minimizes the sum of the squared deviations

$$\sum_{i=1}^{n} ((a + bx_i + cx_i^2) - y_i)^2.$$

The method is the same, but the results are far more complicated.

Quadratic Regression

Quadratic Regression

- The equations to be solved for a, b, and c are

$$\begin{aligned} na + b\sum x + c\sum x^2 &= \sum y, \\ a\sum x + b\sum x^2 + c\sum x^3 &= \sum xy, \\ a\sum x^2 + b\sum x^3 + c\sum x^4 &= \sum x^2 y. \end{aligned}$$

Quadratic Regression

$$a = \frac{(\sum x^2)^2 \sum x^2y - \sum x \sum x^2y \sum x^3 + \sum x \sum x^4 \sum xy + (\sum x^2)^2 \sum xy - \sum x^2 \sum x^3 \sum x^2y + \sum x \sum x^4 \sum y}{(\sum x^2)^3 - 2\sum x \sum x^2 \sum x^3 + n(\sum x^3)^2 + (\sum x)^2 \sum x^4 - n \sum x^2 \sum x^4},$$

$$b = \frac{n \sum x^2y \sum x^3 - \sum x \sum x^2 \sum x^2y + (\sum x^2)^2 \sum xy + \sum x \sum x^4 \sum y - n \sum x^4 \sum xy - \sum x^2 \sum x^3 \sum y}{(\sum x^2)^3 - 2\sum x \sum x^2 \sum x^3 + n(\sum x^3)^2 + (\sum x)^2 \sum x^4 - n \sum x^2 \sum x^4},$$

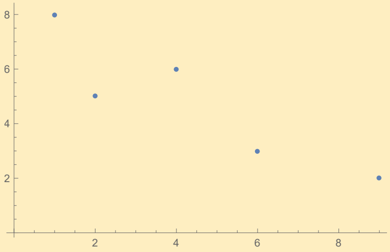
$$c = \frac{(\sum x)^2 \sum x^2y - n \sum x^2 \sum x^2y - \sum x \sum x^2 \sum xy + n \sum x^3 \sum xy + (\sum x^2)^2 \sum y - \sum x \sum x^3 \sum y}{(\sum x^2)^3 - 2\sum x \sum x^2 \sum x^3 + n(\sum x^3)^2 + (\sum x)^2 \sum x^4 - n \sum x^2 \sum x^4}.$$

Quadratic Regression

Example

Find the least-squares regression quadratic function for the data

$$(1, 8), (2, 5), (4, 6), (6, 3), (9, 2).$$



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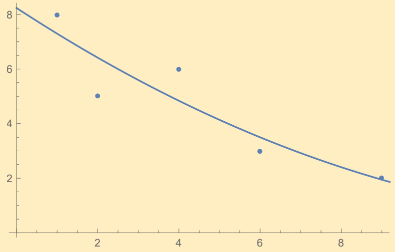

$$y = 0.0302x^2 - 0.9709x + 8.2387$$

Logarithmic Regression

Logarithmic Regression

Logarithmic regression requires finding a logarithmic function

$$y = a + b \ln x$$

that minimizes the sum of the squared deviations

$$\sum_{i=1}^{n} ((a + b \ln x_i) - y_i)^2.$$

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This is the same as linear regression with x replaced by $\ln x$.

Logarithmic Regression

The equations to be solved are

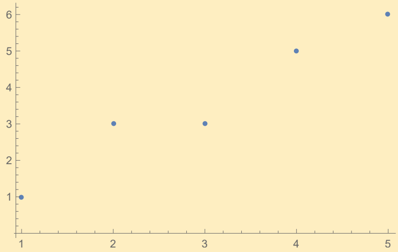
$$na + b\sum \ln x_i = \sum y_i,$$
$$a\sum \ln x_i + b\sum (\ln x_i)^2 = \sum (\ln x_i)y_i.$$

Logarithmic Regression

Example

Find the least-squares regression logarithmic function for the data

$$(1,1),(2,3),(3,3),(4,5),(5,6).$$



Logarithmic Regression

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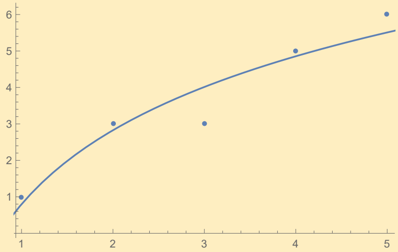

$$y = 0.797 + 2.927 \ln x$$

Power Regression

Power Regression

Power regression requires finding a logarithmic function

$$y = ax^b$$

that minimizes the sum of the squared deviations

$$\sum_{i=1}^{n} (ax_i^b - y_i)^2.$$

Power Regression

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The trick is to apply logarithms and convert the equation into

$$\ln y = \ln a + b \ln x$$

and use logarithmic regression.

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Again, this is linear regression

$$Y = A + bX$$

with y replaced by $\ln y$, y replaced by $\ln y$, and a replaced by $\ln a$.

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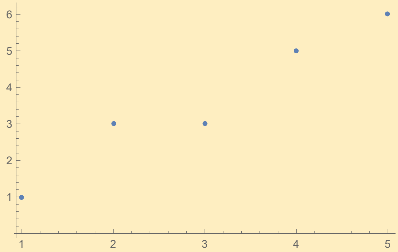
with y replaced by $\ln y$, y replaced by $\ln y$, and a replaced by $\ln a$.
The solution gives A for $\ln a$, so $a = e^A$.

Power Regression

Example

Find the least-squares regression power function for the data

$$(1, 1), (2, 3), (3, 3), (4, 5), (5, 6).$$



Power Regression

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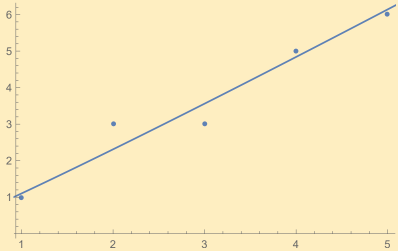

$$y = 1.1036x^{1.0665}$$

Exponential Regression

Exponential Regression

Exponential regression requires finding a logarithmic function

$$y = ab^x$$

that minimizes the sum of the squared deviations

$$\sum_{i=1}^{n} (ab^{x_i} - y_i)^2.$$

Exponential Regression

The trick is again to apply logarithms and convert the equation into

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with x replaced by $\ln x$, a replaced by $\ln a$, and b replaced by $\ln b$.

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The trick is again to apply logarithms and convert the equation into

$$\ln y = \ln a + x \ln b$$

and use logarithmic regression.

Once again, this is linear regression

$$Y = A + Bx$$

with x replaced by $\ln x$, a replaced by $\ln a$, and b replaced by $\ln b$.

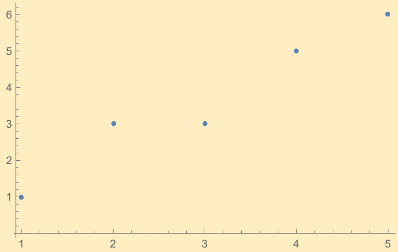
The solution gives A for $\ln a$ and B for $\ln b$, so $a = e^A$ and $b = e^B$.

Exponential Regression

Example

Find the least-squares regression exponential function for the data

$$(1,1),(2,3),(3,3),(4,5),(5,6).$$



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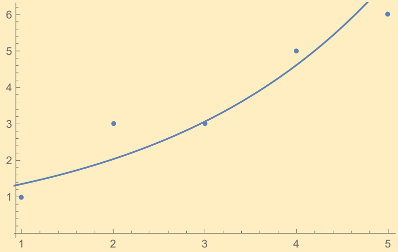
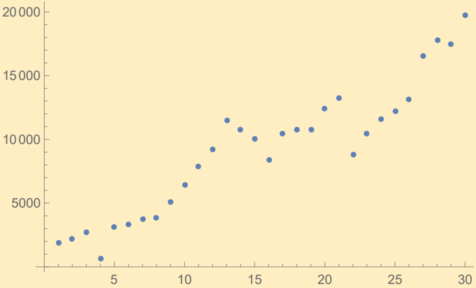


$$y = 0.897(1.506^x)$$

Example

DJIA and Exponential Growth

Fit the various models to the Dow-Jones Industrials year-end average for the last 30 years



The year-end averages

Example

DJIA and Exponential Growth

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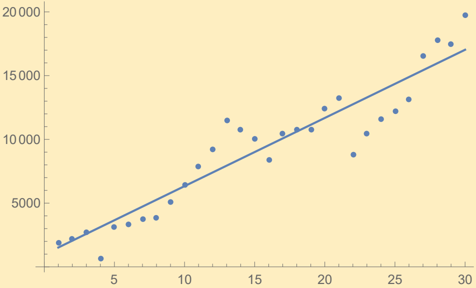

Linear model: $y = 983.7 + 534.87x$

Example

DJIA and Exponential Growth

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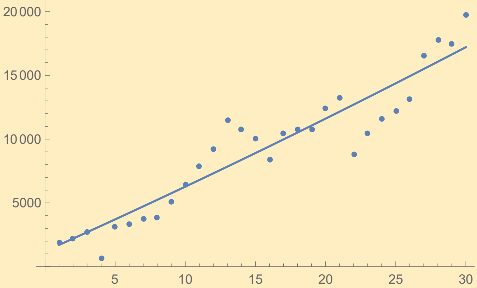

Quadratic model: $y = 1.35x^2 + 493.0x + 1206.9$

Example

DJIA and Exponential Growth

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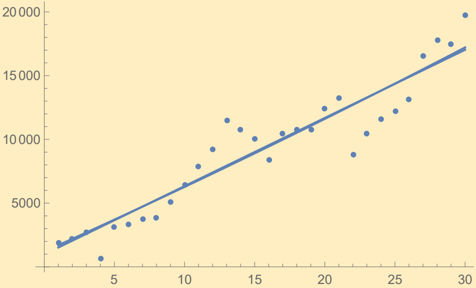
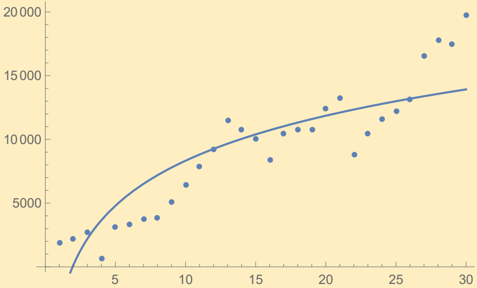


Linear model vs. quadratic model

Example

DJIA and Exponential Growth

Fit the various models to the Dow-Jones Industrials year-end average for the last 30 years



Logarithmic model: $y = -3395.8 + 5091.2 \ln x$

Example

DJIA and Exponential Growth

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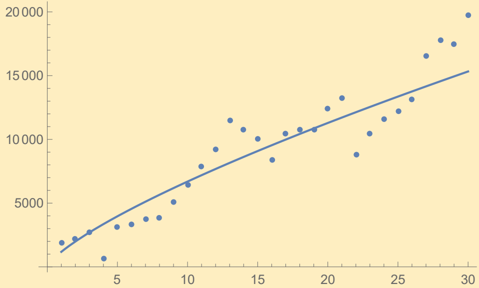

Power model: $y = 1178.1x^{0.7543}$

Example

DJIA and Exponential Growth

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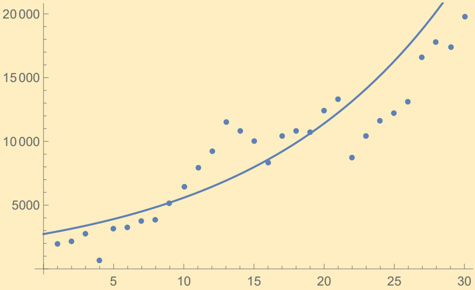

Exponential model: $y = 2734.3(1.074^x)$