

The Least-Squares Regression Line

Lecture 6
Section 1.3

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Objectives

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- The equation of a line.
- Parallel and perpendicular lines.
- Finding the intercepts.
- Least-squares regression line.

Slope

Definition (Slope)

The **slope** of a line is the vertical change divided by the horizontal change (“rise over run”) between any two points (x_1, y_1) and (x_2, y_2) on the line.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-Slope Form

Definition (Point-Slope Form)

The **point-slope form** of the equation of a line is the equation

$$y - y_0 = m(x - x_0)$$

where m is the slope and (x_0, y_0) is a point (any point) on the line.

Slope-Intercept Form

Definition (Slope-Intercept Form)

The **slope-intercept form** of the equation of a line is the equation

$$y = mx + b$$

where m is the slope and b is the y -intercept.

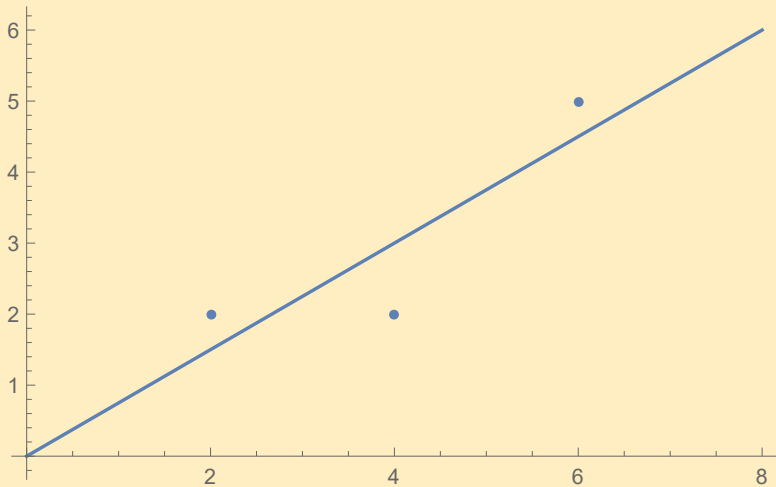
The Least-Squares Regression Line

Definition (The Least-Squares Regression Line)

Given a set of data point $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$, the **least-squares regression line** is the (unique) line that minimizes the sum of the vertical distances from the data points to the line.

Example

Least-Squares Regression Line $y = \frac{3}{4}x$



Calculating the Least-Squares Line

Given the data points $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$, calculate the sums

$$\sum x = x_1 + x_2 + \cdots + x_n,$$

$$\sum y = y_1 + y_2 + \cdots + y_n,$$

$$\sum x^2 = x_1^2 + x_2^2 + \cdots + x_n^2,$$

$$\sum xy = x_1y_1 + x_2y_2 + \cdots + x_ny_n.$$

Then ...

Calculating the Least-Squares Line

The point

$$(\bar{x}, \bar{y})$$

is a **point** on the line, where \bar{x} is the average of the x values and \bar{y} is the average of the y values and the **slope** of the line is given by

$$m = \frac{n \sum xy - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)(\sum x)}.$$

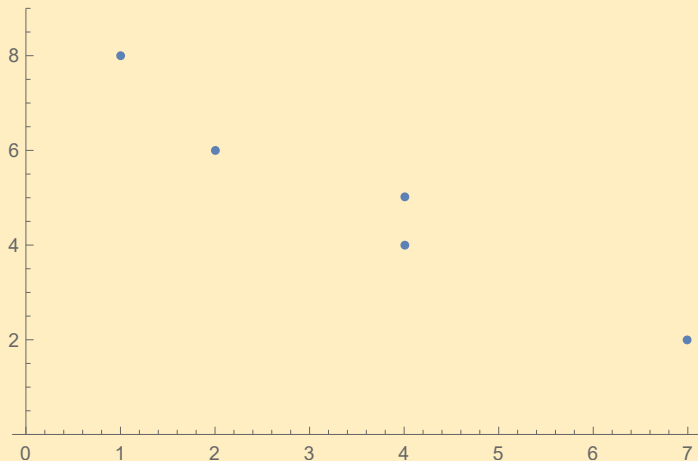
Calculating the Least-Squares Line

Find the least-squares regression line through the points

$$(1, 8), (2, 6), (4, 5), (4, 4), (7, 2).$$

Example

Least-Squares Regression Line



Example

Least-Squares Regression Line

