Wednesday, August 26, 2015

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Problem 53

Problem. Find the derivative of $y = (7x+3)^4$.

Solution. Use the Power Rule and the Chain Rule.

$$\begin{aligned} y' &= 4(7x+3)^3 \cdot \frac{d}{dx}(7x+3) \\ &= 4(7x+3)^3(7) \\ &= 28(7x+3)^3. \end{aligned}$$

Problem 55

Problem. Find the derivative of $y = \dfrac{1}{x^2+4}$.

Solution. Use the special case of the Quotient Rule:

$$\frac{d}{dx}\left(\frac{1}{f(x)}\right) = -\frac{f'(x)}{(f(x))^2}.$$

Then

$$y' = -\frac{2x}{(x^2+4)^2}.$$

Problem 57

Problem. Find the derivative of $y = 5\cos(9x+1)$.

Solution. Use the rule for cosine and the Chain Rule.

$$\begin{aligned} y' &= -5\sin(9x+1) \cdot 9 \\ &= -45\sin(9x+1). \end{aligned}$$

Problem 59

Problem. Find the derivative of $y = \dfrac{x}{2} - \dfrac{\sin 2x}{4}$.

Solution. Use the rule for sine and the Chain Rule.

$$\begin{aligned} y' &= \frac{1}{2} - \frac{(\cos 2x)\cdot 2}{4} \\ &= \frac{1}{2} - \frac{\cos 2x}{2}. \end{aligned}$$

Problem 61

Problem. Find the derivative of $y = x(6x+1)^5$.

Solution. Use the Product Rule, the Power Rule, and the Chain Rule.

$$\begin{aligned} y' &= 1 \cdot (6x+1)^5 + x \cdot 5(6x+1)^4 \cdot 6 \\ &= (6x+1)^5 + 30x(6x+1)^4. \end{aligned}$$

Problem 63

Problem. Find the derivative of $f(x) = \dfrac{3x}{\sqrt{x^2+1}}$.

Solution. Use the Quotient Rule, the Power Rule (for the square root), and the Chain Rule.

$$\begin{aligned} f'(x) &= \frac{3 \cdot \sqrt{x^2+1} - 3x \cdot \frac{d}{dx}(\sqrt{x^2+1})}{x^2+1} \\ &= \frac{3\sqrt{x^2+1} - 3x \cdot \left(\frac{x}{\sqrt{x^2+1}}\right)}{x^2+1} \\ &= \frac{3(x^2+1) - 3x^2}{(x^2+1)\sqrt{x^2+1}} \\ &= \frac{3}{(x^2+1)^{3/2}}. \end{aligned}$$

Problem 65

Problem. Find and evaluate the derivative of $f(x) = \sqrt{1-x^3}$ at $x = -2$.

Solution. The derivative is

$$\begin{aligned} f'(x) &= \frac{1}{2} \cdot (1-x^3)^{-1/2} \cdot (-3x^2) \\ &= -\frac{3x^2}{2\sqrt{1-x^3}}. \end{aligned}$$

Then

$$\begin{aligned} f'(-2) &= -\frac{3(-2)^2}{2\sqrt{1-(-2)^3}} \\ &= -\frac{12}{6} \\ &= -2. \end{aligned}$$

Problem 67

Problem. Find and evaluate the derivative of $f(x) = \dfrac{4}{x^2+1}$ at $x = -1$.

Solution. The derivative is

$$\begin{aligned} f'(x) &= -\frac{4 \cdot 2x}{(x^2+1)^2} \\ &= -\frac{8x}{(x^2+1)^2}. \end{aligned}$$

Then

$$\begin{aligned} f'(-1) &= -\frac{8(-1)}{((-1)^2+1)^2} \\ &= \frac{8}{4} \\ &= 2. \end{aligned}$$

Problem 69

Problem. Find and evaluate the derivative of $f(x) = \dfrac{1}{2}\csc 2x$ at $x = \frac{\pi}{4}$.

Solution. The derivative is

$$\begin{aligned} f'(x) &= \frac{1}{2} \cdot (-\cot 2x \csc 2x) \cdot 2 \\ &= -\cot 2x \csc 2x. \end{aligned}$$

Then

$$\begin{aligned} f'\left(\frac{\pi}{4}\right) &= -\cot\frac{\pi}{2}\csc\frac{\pi}{2} \\ &= -(0)(1) \\ &= 0. \end{aligned}$$

Problem 86

Problem. All edges of a cube are expanding at a rate of 8 centimeters per second. How fast is the surface area changing when each edge is 6.5 centimeters?

Solution. Let x be the side of the cube and let s be the surface area. Then $s = 6x^2$. Now, s and x are both functions of time t, so differentiate with respect to t to get

$$\frac{ds}{dt} = 12x \cdot \frac{dx}{dt}.$$

We are asked to find the rate of change of the surface area, that is, $\frac{ds}{dt}$ when $x = 6.5$, given that the rate of change of x, that is, $\frac{dx}{dt}$, is 8. Thus, we substitute $x = 6.5$ and $\frac{dx}{dt} = 8$ into the equation to get

$$\frac{ds}{dt} = 12 \cdot 6.5 \cdot 8 = 624.$$

Problem 87

Problem. A rotating beacon is located 1 ilometer off a straight shoreline. The bacon rotates at a rate of 3 revolutions per minute. How fast does the beam of light appear to be moving to a viewer whoe is $\frac{1}{2}$ kilometer down the shoreline?

Solution. Let x be the distance along the shoreline and let θ be the angle (as shown in the diagram). First, we need an equation (that is, a function) relating x and θ. Using the right triangle with sides 1 and x, we see that

$$\tan\theta = x.$$

Now we can differentiate with respect to time and get

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{dx}{dt}.$$

We are asked to find $\frac{dx}{dt}$. The value of $\frac{d\theta}{dt}$ is given as 3 revolutions per minute, which equals 6π radians per minute. So, $\frac{d\theta}{dt} = 6\pi$. That leaves $\sec\theta$. From the right triangle, we see that

$$\sec\theta = \sqrt{x^2+1}.$$

Therefore, when $x = \frac{1}{2}$, we have $\sec\theta = \sqrt{\frac{5}{4}}$. Substituting these values, we get

$$\left(\sqrt{\frac{5}{4}}\right)^2 \cdot 6\pi = \frac{dx}{dt}$$
$$\frac{dx}{dt} = \frac{15\pi}{4}.$$

Problem 88

Problem. A sandbag is dropped from a balloon at a height of 60 meters when the angle of elevation to the sun is 30°. The position of the sandbag is

$$s(t) = 60 - 4.9t^2.$$

Find the rate at which the shadow of the sandbag is traveling along the ground when the sandbag is at a height of 34 meters.

Solution. Let x be the distance to the shadow of the sandbag (the base of the triangle). We need an equation relating x and $s(t)$. The sides of the right triangle are x and $s(t)$ and the angle is $30°$ (constant). Therefore,

$$\frac{s(t)}{x} = \tan 30° = \frac{1}{\sqrt{3}},$$

so

$$x = s(t)\sqrt{3}.$$

Differentiate to get

$$\begin{aligned}\frac{dx}{dt} &= s'(t)\sqrt{3}\\ &= (-9.8t)\sqrt{3}.\end{aligned}$$

The sandbag will be 35 meters above the ground when $60 - 4.9t^2 = 35$. Solve that equation to get $t = \frac{5}{\sqrt{4.9}} = 2.259$. Thus,

$$\begin{aligned}\frac{dx}{dt} &= (-9.8)(2.259)\sqrt{3}\\ &= -38.34.\end{aligned}$$

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Problem 15

Problem. Find the indefinite integral $\int (x^{3/2} + 2x + 1)\, dx$.

Solution. Use the Power Rule for integration.

$$\int (x^{3/2} + 2x + 1)\, dx = \frac{2}{5}x^{5/2} + x^2 + x + C.$$

Problem 25

Problem. Find the indefinite integral $\int (5\cos x + 4\sin x)\, dx$.

Solution. Use the rules for $\sin x$ and $\cos x$.

$$\begin{aligned}\int (5\cos x + 4\sin x)\, dx &= 5(\sin x) + 4(-\cos x) + C\\ &= 5\sin x - 4\cos x + C.\end{aligned}$$

Problem 53

Problem. A ball is thrown vertically upward from a height of 6 feet with an initial velocity of 60 feet per second. How high will the ball go?

Solution. Let $x(t)$ be the height of the ball at time t. The gravitational acceleration is $a(t) = -32$, so $x''(t) = -32$. Then integrate to get

$$x'(t) = -32t + v_0.$$

At time $t = 0$, the velocity is 60, so $v_0 = 60$ and we have $x'(t) = -32t + 60$. The ball will reach its highest point when its velocity is 0, so solve $x'(t) = 0$ to get $t = \frac{15}{8}$. Now integrate again to get

$$x(t) = -16t^2 + 60t + x_0.$$

The height at time $t = 0$ is 6, so $x_0 = 6$ and we have $x(t) = -16t^2 + 60t + 6$. Now let $t = \frac{15}{8}$ to find the height when the velocity is 0.

$$\begin{aligned} x\left(\tfrac{15}{8}\right) &= -16\left(\frac{15}{8}\right)^2 + 60\left(\frac{15}{8}\right) + 6 \\ &= \frac{249}{4} \\ &= 62.25. \end{aligned}$$