Math 142 **Test #1** **Fall 2011**

1. (35 pts)

   (a) (7 pts) $f'(x) = 3e^{3x-1}$.

   (b) (7 pts) Use the properties of logarithms first to get $f(x) = \ln(2x+1) - \ln(x+1)$. Then differentiate and get $f'(x) = \dfrac{2}{2x+1} - \dfrac{1}{x+1}$.

   (c) (7 pts) You should memorize this one: $f'(x) = 2\sec^2 2x$. Or you can use $\tan 2x = \dfrac{\sin 2x}{\cos 2x}$ and then use the quotient rule.

   (d) (7 pts) $f'(x) = \dfrac{1}{1+(2x)^2} \cdot (2) = \dfrac{2}{1+4x^2}$.

   (e) (7 pts) $f'(x) = (2x+1)10^{x^2+x}\ln 10$.

2. (40 pts)

   (a) (8 pts) First use long division to get that $\dfrac{x^2-4x+4}{x-3} = x - 1 + \dfrac{1}{x-3}$. Then integrate.

$$\begin{aligned}\int_4^6 \left(x - 1 + \frac{1}{x-3}\right)\,dx &= \left[\frac{1}{2}x2 - x + \ln|x-3|\right]_4^6 \\ &= (18 - 6 + \ln 3) - (8 - 4 + \ln 1) \\ &= 8 + \ln 3.\end{aligned}$$

   (b) (8 pts) You should memorize that $\int \tan x\,dx = \ln|\sec x| + C$. So make the substitution $u = 5x$, $du = 5\,dx$. We get

$$\begin{aligned}\int \tan 5x\,dx &= \frac{1}{5}\int \tan u\,du \\ &= \frac{1}{5}\ln|\sec u| + C \\ &= \frac{1}{5}\ln|\sec 5x| + C.\end{aligned}$$

   (c) (8 pts) Divide each term in the numerator by e^{2x} to get $\dfrac{2+e^x}{e^{2x}} = 2e^{-2x} + e^{-x}$. Then

$$\int \left(2e^{-2x} + e^{-x}\right)\,dx = -e^{-2x} - e^{-x} + C.$$

   (d) (8 pts) Let $u = 2x$, $du = 2dx$. Then

$$\begin{aligned}\int \frac{2}{\sqrt{1-4x^2}}\,dx &= \int \frac{1}{\sqrt{1-u^2}}\,du \\ &= \arcsin u + C \\ &= \arcsin 2x + C.\end{aligned}$$

(e) (8 pts) Use the formula that $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C$. Then

$$\begin{aligned}\int_{-2}^{2} \frac{1}{4+x^2}\,dx &= \left[\frac{1}{2}\arctan\frac{x}{2}\right]_{-2}^{2} \\ &= \frac{1}{2}\arctan 1 - \frac{1}{2}\arctan(-1) \\ &= \frac{1}{2}\cdot\frac{\pi}{4} - \frac{1}{2}\cdot\left(-\frac{\pi}{4}\right) \\ &= \frac{\pi}{4}.\end{aligned}$$

3. (5 pts) Use the substitution $u = 2 + e^x$, $du = e^x\,dx$. Then $u(0) = 3$ and $u(\ln 2) = 4$ and $e^x = u - 2$. So

$$\begin{aligned}\int_{0}^{\ln 2} \frac{e^{2x}}{2+e^x}\,dx &= \int_3^4 \frac{u-2}{u}\,du \\ &= \int_3^4 \left(1 - \frac{2}{u}\right)\,du \\ &= [u - 2\ln|u|]_3^4 \\ &= (4 - 2\ln 4) - (3 - 2\ln 3) \\ &= 1 - 2\ln 4 + 2\ln 3.\end{aligned}$$

4. (8 pts)

(a) (2 pts) Write the function f as $y = \sqrt{8 - x^3}$. Switch x and y and solve for y:

$$\begin{aligned}x &= \sqrt{8 - y^3} \\ x^2 &= 8 - y^3 \\ y^3 &= 8 - x^2 \\ y &= (8 - x^2)^{1/3}.\end{aligned}$$

So $f^{-1}(x) = (8 - x^2)^{1/3}$.

(b) (2 pts) We are told that the domain of f is $x \le 2$ and that the function is decreasing on its domain. We find that $f(2) = 0$. Therefore, 0 is its largest value and so the range of f is $[0, \infty)$.

(c) (2 pts) The domain of f^{-1} is the range of f and the range of f^{-1} is the domain of f. Therefore, the domain of f^{-1} is $[0, \infty)$ and the range of f^{-1} is $(-\infty, 2]$.

(d) (2 pts) The derivative of f^{-1} is $(f^{-1})'(x) = \frac{1}{3}(8 - x^2)^{-2/3}(-2x)$.

5. (2 pts) The function takes on the value 0 at $x = 0$ and again at $x = 1$. Therefore, it fails the horizontal line test.

6. (10 pts)

(a) Because the principle doubles in 12 years, we can write the function as $P(t) = 1000 \cdot 2^{t/12}$. Or we can begin with the form $P(t) = 1000e^{kt}$ and solve for k. Let $t = 12$. Then

$$\begin{aligned} P(12) &= 1000e^{12k} \\ 2000 &= 1000e^{12k} \\ 2 &= e^{12k} \\ \ln 2 &= 12k \\ k &= \frac{1}{12}\ln 2 \\ &= 0.05776. \end{aligned}$$

So the function is $P(t) = 1000e^{0.05776t}$.

(b) Let $t = 6$ and compute either

$$\begin{aligned} P(6) &= 1000 \cdot 2^{6/12} \\ &= 1000\sqrt{2} \\ &= 1414.21, \end{aligned}$$

or

$$\begin{aligned} P(6) &= 1000e^{(0.05776)(6)} \\ &= 1000e^{0.34657} \\ &= 1000(1.41421) \\ &= 1414.21. \end{aligned}$$

(c) Solve the equation $P(t) = 5000$ for t. Solve either

$$\begin{aligned} 1000 \cdot 2^{t/12} &= 5000 \\ 2^{t/12} &= 5 \\ \left(\frac{t}{12}\right)\ln 2 &= \ln 5 \\ t &= 12\left(\frac{\ln 5}{\ln 2}\right) \\ &= 27.86, \end{aligned}$$

or

$$\begin{aligned} 1000e^{0.05776t} &= 5000 \\ e^{0.05776t} &= 5 \\ 0.05776t &= \ln 5 \\ t &= \frac{\ln 5}{0.05776} \\ &= 27.86. \end{aligned}$$

(d) Differentiate to get either $P'(t) = 1000 \cdot 2^{t/12} \left(\frac{1}{12}\right) \ln 2 = 57.76 \cdot 2^{t/12}$ or $P'(t) = 1000e^{0.05776}(0.05776) = 57.76e^{0.05776t}$.