

You must show all your work to receive full credit, especially when differentiating and integrating functions. If you are using a TI-89, you may use it to check answers, but you must show how you found derivatives and integrals. Some of these problems can be solved by methods other than the ones specified. You may use other methods to receive partial credit or to check your answers.

1. (22 pts) Find the following limits, if they exist.

(a) (10 pts) $\lim_{x \rightarrow 0} \frac{\arctan x}{x^2}$

(b) (12 pts) $\lim_{x \rightarrow 0^-} (1+x)^{\csc x}$

2. (10 pts) Find the indefinite integral

$$\int \cos^3 x \, dx.$$

3. (15 pts) Use a trigonometric substitution to find the indefinite integral

$$\int \frac{x^3}{\sqrt{x^2 + 16}} \, dx.$$

4. (18 pts) Use integration by parts to find the definite integral

$$\int_0^{\pi/4} e^x \sin x \, dx.$$

5. (24 pts) Use the method of partial fractions to find the following indefinite integrals.

(a) (12 pts) $\int \frac{x}{(x-2)^3} \, dx$

(b) (12 pts) $\int \frac{x^2 - 2x - 1}{(x+1)(x^2+1)} \, dx$

6. (12 pts) Evaluate the following improper integral

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2} \, dx$$