

You must show all your work to receive full credit, especially when differentiating and integrating functions. If you are using a TI-89, you may use it to check answers, but you must show how you found derivatives and integrals.

1. (35 pts) Find the derivative of each of the following functions.

(a)  $f(x) = e^x$

(b)  $f(x) = \frac{e^x}{1 + e^x}$

(c)  $f(x) = x^2 \ln(x^2 + 1)$

(d)  $f(x) = \log_2 x$

(e)  $f(x) = \arctan 3x$

2. (8 pts) Use logarithmic differentiation to find the derivative of

$$y = \frac{(2x + 1)^3}{\sqrt{x^2 - 1}}.$$

3. (35 pts) Find each of the following definite or indefinite integrals.

(a)  $\int_2^3 \frac{2x + 3}{x^2 + 3x - 9} dx$

(b)  $\int_2^4 \frac{x^2}{x - 1} dx$

(c)  $\int \tan x dx$

(d)  $\int \frac{\ln x}{x} dx$

(e)  $\int e^x \sqrt{e^x + 1} dx$

4. (6 pts) Let  $f(x) = \sqrt{x - 4}$ ,  $x \geq 4$ .

(a) Find  $f^{-1}(x)$ .

(b) Find the domain and range of  $f^{-1}(x)$ .

5. (6 pts) Compute the values of the following expressions.

(a)  $\arcsin 0.5$

(b)  $\arcsin(\sin \pi)$

(c)  $\cos(\arctan 1)$

6. (10 pts) The overall growth rate of the U.S. population (expressed as a percentage) is 0.9%, meaning that each year the population increases by 0.9% over the previous year. Also, in mid-2007, the U.S. population was approximately 301,140,000 people. Assuming an exponential model of population growth, this leads to the equation

$$P(t) = 301140000e^{0.00896t},$$

where  $t$  is time in years, with  $t = 0$  representing mid-2007, and  $P(t)$  is the population at time  $t$ . Use this model to answer the following questions.

- (a) What will be the U.S. population in mid-2017?
- (b) Find a function for the rate of growth of the U.S. population (expressed in people/year).
- (c) At what rate was the U.S. population growing in mid-2007 (in people/year)?
- (d) At what rate will the U.S. population be growing in mid-2017 (in people/year)?
- (e) When will the size of the U.S. population be double what it was in mid-2007?