Using Definitions

Lecture 2

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Outline

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1. Definitions
2. Typesetting Mathematics
   - The LaTeX File
   - Mathematical Expressions
   - Environments
   - Integers, Real Numbers, Etc.
3. Assignment

Definitions

- The purpose of a definition is to associate a word with a concept.
- Often, for conciseness, a theorem will use the terminology rather than describing the concept.
- To prove the theorem, we must apply the definitions of the terms.

Definitions

Theorem

If a function f is differentiable at $x = c$, then f is continuous at $x = c$.

- The statement uses the terms *function*, *differentiable*, and *continuous*.

Definition (Continuous)

A function f is continuous at $x = c$ if

$$\lim_{x \to c} f(x) = f(c).$$

Definition (Differentiable)

A function f is differentiable at $x = c$ if

$$\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

exists.

Even and Odd Integers

Definition (Even Integer)

An integer n is even if there exists an integer k such that $n = 2k$.

Definition (Odd Integer)

An integer n is odd if there exists an integer k such that $n = 2k + 1$.

Definition (Multiple)

An integer a is a multiple of an integer b if there exists an integer c such that $a = bc$.

Even and Odd Integers

Theorem

The sum of any two consecutive odd integers is a multiple of 4.

Proof.

□

Even and Odd Integers

Theorem

The sum of any two consecutive odd integers is a multiple of 4.

Proof.

- Let n be an odd integer.

□

Even and Odd Integers

Theorem

The sum of any two consecutive odd integers is a multiple of 4.

Proof.

- Let n be an odd integer. Then $n = 2k + 1$ for some integer k.

□

Even and Odd Integers

Theorem

The sum of any two consecutive odd integers is a multiple of 4.

Proof.

- Let n be an odd integer. Then $n = 2k + 1$ for some integer k.
- The next odd integer is $n + 2$, so $n + 2 = 2k + 3$.

□

Even and Odd Integers

Theorem

The sum of any two consecutive odd integers is a multiple of 4.

Proof.

- Let n be an odd integer. Then $n = 2k + 1$ for some integer k.
- The next odd integer is $n + 2$, so $n + 2 = 2k + 3$.
- Then the sum of the consecutive odd integers is

$$n + (n + 2)$$

□

Even and Odd Integers

Theorem

The sum of any two consecutive odd integers is a multiple of 4.

Proof.

- Let n be an odd integer. Then $n = 2k + 1$ for some integer k.
- The next odd integer is $n + 2$, so $n + 2 = 2k + 3$.
- Then the sum of the consecutive odd integers is

$$n + (n + 2) = (2k + 1) + (2k + 3)$$

□

Even and Odd Integers

Theorem

The sum of any two consecutive odd integers is a multiple of 4.

Proof.

- Let n be an odd integer. Then $n = 2k + 1$ for some integer k.
- The next odd integer is $n + 2$, so $n + 2 = 2k + 3$.
- Then the sum of the consecutive odd integers is

$$\begin{aligned} n + (n + 2) &= (2k + 1) + (2k + 3) \\ &= 4k + 4 \end{aligned}$$

□

Even and Odd Integers

Theorem

The sum of any two consecutive odd integers is a multiple of 4.

Proof.

- Let n be an odd integer. Then $n = 2k + 1$ for some integer k.
- The next odd integer is $n + 2$, so $n + 2 = 2k + 3$.
- Then the sum of the consecutive odd integers is

$$\begin{aligned} n + (n+2) &= (2k+1) + (2k+3) \\ &= 4k + 4 \\ &= 4(k+1) \end{aligned}$$

□

Even and Odd Integers

Theorem

The sum of any two consecutive odd integers is a multiple of 4.

Proof.

- Let n be an odd integer. Then $n = 2k + 1$ for some integer k.
- The next odd integer is $n + 2$, so $n + 2 = 2k + 3$.
- Then the sum of the consecutive odd integers is

$$\begin{aligned} n + (n + 2) &= (2k + 1) + (2k + 3) \\ &= 4k + 4 \\ &= 4(k + 1) \end{aligned}$$

which is a multiple of 4.

□

Even and Odd Integers

Theorem

The sum of any two consecutive even integers is not a multiple of 4.

Proof.

□

Even and Odd Integers

Theorem

The product of three consecutive integers is a multiple of 6, but it is not necessarily a multiple of 12.

Proof.

□

Outline

LaTeX Modes

- LaTeX is a mathematical typesetting system.
- This slideshow was written in LaTeX.
- There are two modes: text and math.
- The dollar sign ($) is used to toggle between the modes.

The LaTeX File

- A LaTeX file uses the `.tex` extension.
- The first line is

```
\documentclass[12pt]{article}
```

- This is followed by a preamble, which we will discuss later.
- The body of the document is placed between the delimiters

```
\begin{document}
     :
\end{document}
```

Mathematical Expressions

- The basic operators.
  - Addition: +
  - Subtraction: −
  - Multiplication: `\cdot`(·) or `\times` (×)
  - Exponentiation: `^{`*exp*`}`
  - Division: / or `\div`(÷) or `\frac{`*num*`}{`*den*`}`
- Extendible grouping symbols.
  - Parentheses: `\left(...\right)`
  - Square brackets: `\left[...\right]`
  - Curly braces: `\left\{...\right\}`

Mathematical Expressions

Example (Mathematical Expressions)

The LaTeX expression

```
\left(\frac{2x-1}{x+1}\right)^{n+1}
```

is rendered as

$$\left(\frac{2x-1}{x+1}\right)^{n+1}$$

Mathematical Expressions

Example (Mathematical Expressions)

The LaTeX expression

```
(n+1)^{\frac{2x-1}{x+1}}
```

is rendered as

$$(n+1)^{\frac{2x-1}{x+1}}$$

Typefaces

Example (Typefaces)

- The standard typefaces, in text mode.
  - Boldface: `\textbf{...}`
  - Italicized: `\textit{...}`
- The standard typefaces, in math mode.
  - Boldface: `\mathbf{...}`
  - Not italicized: `\text{...}`

The `align` Environment

- To display a series of equations, all aligned with the equal sign (or any symbol of your choice), use the `align` environment.
- The alignment character is the ampersand `&`.
- Use \\ to start a new line.
- An environment is delimited by `\begin{`*env-name*`}` and `\end{`*env-name*`}`.
- LaTeX provides many kinds of environments.

The `align` Environment

- For example,

```
\begin{align*}
y &= (x+1)^2-1 \\
&= (x^2+2x+1)-1 \\
&= x^2+2x \\
&= x(x+2).
\end{align*}
```

will be rendered as

$$\begin{aligned} y &= (x+1)^2-1 \\ &= (x^2+2x+1)-1 \\ &= x^2+2x \\ &= x(x+2). \end{aligned}$$

The `align` Environment

- The * means "do not number the equations."
- If we leave it off, then we get

$$
\begin{align}
y &= (x+1)^2 - 1 \\
&= (x^2 + 2x + 1) - 1 \\
&= x^2 + 2x \\
&= x(x+2).
\end{align}
$$

Integers, Real Numbers, Etc.

- The whole numbers are $1, 2, 3, \ldots$.
- The integers are the whole numbers, their negatives, and zero:

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

- The rational numbers are the fractions of integers:

$$\left\{\frac{a}{b} \middle| a, b \text{ are integers}, b \neq 0\right\}$$

Integers, Real Numbers, Etc.

- The real numbers are rational numbers together with the irrational numbers.
  - The real numbers are all numbers that can be expressed in terminating or nonterminating decimal form.
- The complex numbers are the numbers of the form $a + bi$, where a and b are real numbers and $i^2 = -1$.

Standard Symbols

- The standard symbols are
  - $\mathbb{N}$ = the whole numbers.
  - $\mathbb{Z}$ = the integers.
  - $\mathbb{Q}$ = the rational numbers.
  - $\mathbb{R}$ = the real numbers.
  - $\mathbb{C}$ = the complex numbers.
- Use `\mathbb{}` to create that type face.

Outline

Assignment

Homework

- Presentation – Chapter 1:
- Written – Chapter 1: