

# Logical Equivalence

## Lecture 2 Section 2.1

Robb T. Koether

Hampden-Sydney College

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1 Logical Equivalence

2 DeMorgan's Laws

3 Tautologies

4 Assignment

# Outline

- 1 Logical Equivalence
- 2 DeMorgan's Laws
- 3 Tautologies
- 4 Assignment

# Logical Equivalence

$p$	$q$	$(p \wedge q) \vee (\sim p \wedge \sim q)$	$(p \vee \sim q) \wedge (\sim p \vee q)$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

- Two statements are **logically equivalent** if they have the same truth values for all combinations of truth values of their variables.
- $(p \wedge q) \vee (\sim p \wedge \sim q) \equiv (p \vee \sim q) \wedge (\sim p \vee q)$

# Example: Logical Equivalence

- Let  $p$  be “HSC is in North Carolina.”
- Let  $q$  be “UVA is in Virginia.”
- Write the statement corresponding to

$$(p \wedge q) \vee (\sim p \wedge \sim q).$$

- Write the statement corresponding to

$$(p \vee \sim q) \wedge (\sim p \vee q).$$

- Do they express exactly the same idea?
- What idea do they express?

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# DeMorgan's Laws

- DeMorgan's Laws:

$$\sim (p \wedge q) \equiv (\sim p) \vee (\sim q)$$

$$\sim (p \vee q) \equiv (\sim p) \wedge (\sim q)$$

# DeMorgan's Laws

- If it is not true that

$$x > 5 \text{ and } x < 10,$$

then it is true that

$$x \leq 5 \text{ or } x \geq 10.$$

- What about the statement

$$x \leq 5 \text{ and } x \geq 10?$$



# DeMorgan's Laws

- If it is not true that

```
i < size && value != array[i]
```

then it is true that

```
i >= size || value == array[i]
```

# DeMorgan's Laws

- Write a **while** loop that searches an array `arr` of size `size` for the value `value`.
- Write a **repeat-until** (not a C structure) loop that searches an array `arr` of size `size` for the value `value`.

# DeMorgan's Laws

- Write the negations of the following expressions.
  - $\sim p \vee q$
  - $p \wedge q \wedge r$
  - $(p \vee q) \wedge \sim (p \wedge q)$
  - $p \text{ NAND } q$

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# Tautologies and Contradictions

- A **tautology** is a statement that is logically equivalent to T.
- It is a logical form that is true for all values of its variables.
- A **contradiction** is a statement that is logically equivalent to F.
- It is a logical form that is false for all values of its variables.

# Tautologies and Contradictions

- Some tautologies:

$$p \vee \sim p$$

$$p \vee \sim q \vee (\sim p \wedge q)$$

$$p \vee (\sim p \wedge \sim r) \vee \sim p$$

- Some contradictions:

$$p \wedge \sim p$$

$$p \wedge q \wedge (\sim p \vee \sim q)$$

$$(p \wedge \sim q) \wedge (q \wedge \sim r)$$

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# Assignment

## Assignment

- Read Section 2.1, pages 30 - 36.
- Exercises 16, 19, 21, 23, 25, 28, 31, 33, 42, 46, page 37.