Conditional Statements

Lecture 3
Section 2.2

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Fri, Jan 17, 2014

1. Conditional Statements
2. The Contrapositive
3. Other Operators
   - The Biconditional
   - The Exclusive-Or
   - The NAND Operator
   - The NOR Operator
4. Assignment

Outline

1. Conditional Statements
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Conditional Statements

- A conditional statement is a statement of the form

$$p \to q.$$

  - p is the hypothesis.
  - q is the conclusion.
- Read $p \to q$ as "p implies q" or "if p, then q."
- The idea is that the truth of p implies the truth of q (but nothing more).

Example: Conditional Statements

- Which of the following scenarios are consistent with the statement "If I win the lottery, then I will buy a Mercedes Benz."

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  Scenario 1: I win the lottery and I buy a Mercedes Benz.

Example: Conditional Statements

- Which of the following scenarios are consistent with the statement “If I win the lottery, then I will buy a Mercedes Benz.”

  Scenario 1: I win the lottery and I buy a Mercedes Benz.

  Scenario 2: I win the lottery and I do not buy a Mercedes Benz.

Example: Conditional Statements

- Which of the following scenarios are consistent with the statement "If I win the lottery, then I will buy a Mercedes Benz."

Scenario 1: I win the lottery and I buy a Mercedes Benz.

Scenario 2: I win the lottery and I do not buy a Mercedes Benz.

Scenario 3: I do not win the lottery and I do not buy a Mercedes Benz.

Example: Conditional Statements

- Which of the following scenarios are consistent with the statement "If I win the lottery, then I will buy a Mercedes Benz."

  Scenario 1: I win the lottery and I buy a Mercedes Benz.
  Scenario 2: I win the lottery and I do not buy a Mercedes Benz.
  Scenario 3: I do not win the lottery and I do not buy a Mercedes Benz.
  Scenario 4: I do not win the lottery and I buy a Mercedes Benz.

Truth Table for the Conditional

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

- $p \to q$ is true if p is false *or* q is true.
- $p \to q$ is false if p is true *and* q is false.
- Thus, $p \to q$ is logically equivalent to $\sim p \lor q$.

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The Contrapositive

- The contrapositive of $p \to q$ is $\sim q \to \sim p$.
- The statements $p \to q$ and $\sim q \to \sim p$ are logically equivalent. (Prove it!)

The Converse and the Inverse

$p \to q$

$q \to p$

$\sim p \to \sim q$

$\sim q \to \sim p$

Four conditionals.

The Converse and the Inverse

$p \to q$ $\xleftrightarrow{\text{Converses}}$ $q \to p$

$\sim p \to \sim q$ $\xleftrightarrow{\text{Converses}}$ $\sim q \to \sim p$

The converse of $p \to q$ is $q \to p$

The Converse and the Inverse

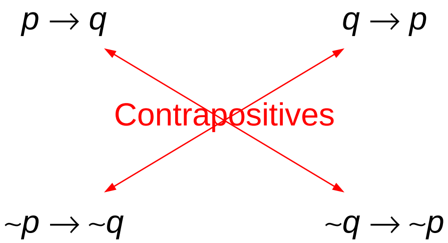
$p \to q$ $\qquad$ $q \to p$

Inverses $\qquad$ Inverses

$\sim p \to \sim q$ $\qquad$ $\sim q \to \sim p$

The inverse of $p \to q$ is $\sim p \to \sim q$

The Converse and the Inverse



The contrapositive of $p \to q$ is $\sim q \to \sim p$

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The Biconditional

- The biconditional of p and q is denoted $p \leftrightarrow q$.
- Read $p \leftrightarrow q$ as "p if and only if q."
- $p \leftrightarrow q$ is logically equivalent to

$$(p \to q) \wedge (q \to p).$$

- It is also logically equivalent to

$$(\sim p \vee q) \wedge (\sim q \vee p)$$

and

$$(p \wedge q) \vee (\sim p \wedge \sim q).$$

The Biconditional

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

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Exclusive-Or

- The exclusive-or of p and q is denoted $p \oplus q$.
- $p \oplus q$ means “one or the other, but not both.”
- $p \oplus q$ is logically equivalent to

$$(p \vee q) \wedge \sim (p \wedge q)$$

and

$$(p \wedge \sim q) \vee (\sim p \wedge q)$$

and

$$\sim (p \leftrightarrow q).$$

Exclusive-Or

p	q	$p \leftrightarrow q$
T	T	F
T	F	T
F	T	T
F	F	F

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The NAND Operator

p	q	$p \mid q$
T	T	F
T	F	T
F	T	T
F	F	T

- The NAND of p and q is denoted $p \mid q$.
- The operator $\mid$ is also called the Scheffer stroke.

The NAND Operator

- The statement $p \mid q$ means "not both p and q."
- $p \mid q$ is logically equivalent to $\sim (p \wedge q)$.

The NAND Operator

- The three basic operators (and, or, not) may be defined in terms of NAND.

$$\sim p \equiv p \mid p.$$
$$p \wedge q \equiv (p \mid q) \mid (p \mid q).$$
$$p \vee q \equiv (p \mid p) \mid (q \mid q).$$

- Prove it!

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The NOR Operator

p	q	$p \downarrow q$
T	T	F
T	F	F
F	T	F
F	F	T

- The NOR of p and q is denoted $p \downarrow q$.
- The operator $\downarrow$ is also called the Pierce arrow.

The NOR Operator

- The statement $p \downarrow q$ means "neither p nor q."
- $p \downarrow q$ is logically equivalent to $\sim (p \vee q)$.

The NOR Operator

- The three basic operators (and, or, not) may be defined in terms of NOR.

$$\sim p \equiv p \downarrow p.$$
$$p \vee q \equiv (p \downarrow q) \downarrow (p \downarrow q).$$
$$p \wedge q \equiv (p \downarrow p) \downarrow (q \downarrow q).$$

- Prove it!

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- Read Section 2.2, pages 39 - 48.
- Exercises 2, 5, 6, 12, 13, 17, 18, 37, 41, 42, 46, page 49.